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De Broglie's Hypothesis

In 1925, Louis de Broglie assumed the existence of a natural symmetry in nature and proposed that the dual character exhibited by photons should equally apply to all material particles.

De Broglie hypothesized that a particle moving with a speed exhibits wave like properties. The wave associated with the moving particle is called matter-wave. The wavelength λ and the frequency ν of the wave associated with a particle of momentum p and energy E are

$$\lambda = \frac{h}{p} \quad \text{and} \quad \nu = \frac{E}{h} \tag{1}$$

Experimental Verification

In 1927, C. J. Davisson and L. H. Germer performed an experiment to verify de Broglie's hypothesis. The experimental setup used is shown in Figure-1.

Electrons from a heated filament are accelerated by a variable voltage V

These electrons emerge through a small hole in the anode with a kinetic energy

$$E_k = eV$$

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The beam then strikes a single crystal of nickel, and the intensity of the scattered beam can be measured for different angles ϕ and various voltages V.

If the propagation of the beam is particle-like, we may expect that the intensity I of the scattered beam will have a smooth monotonic dependence of both ϕ and V because only elastic collisions with the atoms of the crystal are involved.

Thus, if we consider the plane of atoms in the crystal to act like a wall, the incident particles will carom off so that the incoming angle (angle of incidence) is equal to the outgoing angle (angle of reflection) as shown in Figure-2

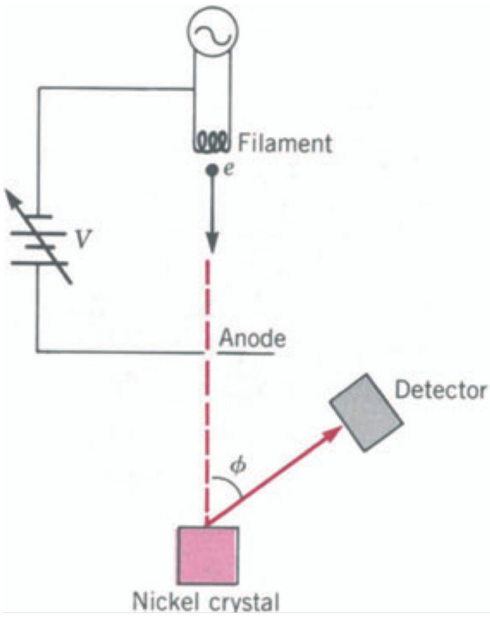

Figure 1: Schematic of the apparatus used in the Davisson-Germer experiment. Electrons from a heated filament are accelerated through a variable potential difference V . They strike a nickel crystal, and the number of electrons scattered at a given angle ϕ is measured as a function of the accelerating voltage. The experiment can be repeated for different values of the angle ϕ.

Then for any incoming angle we will observe the same number of particles caroming off at a similar angle regardless of their velocity and thus of the potential V through which they have been accelerated. On the other hand, if the incoming electrons are not particles but are actually waves, we would expect a diffraction effect like the one observed with X rays when the Bragg scattering condition is satisfied, that is, when

$$n\lambda = 2d \sin\theta$$

From X-ray studies the physicists knew that the spacing between one set of atomic planes of nickel is 0.91Å. For X rays of wavelength 1.65Å the angle of incidence to the plane of atoms that satisfies equation $n\lambda = 2d \sin\theta$ for first-order diffraction ($n = 1$) is

$$\sin\theta = \frac{1.65\ \text{Å}}{2 \times 0.91\ \text{Å}} = 0.907$$
$$\theta = 65^\circ$$

It is seen in Fig. 2 that the angle ϕ between the two beams is defined as $2\theta + \phi = 180^\circ$. The Bragg angle θ is 65° and therefore $\phi = 50^\circ$. This was the angle selected for the experimental arrangement in Fig. 1. Now de Broglie's hypothesis states that the wavelength of a particle is a function of its momentum p. The momentum is related to the kinetic energy of the particle and the kinetic energy depends on its accelerating voltage through the relation $1/2\ mv^2 = eV$. The experimental results of Davisson and Germer are shown in Fig. 3. The angle ϕ between the incoming and the scattered beams of electrons was set at 50°. The accelerating voltage was increased, and the intensity I of the reflected beam was measured for different values of this voltage. A maximum was observed when the voltage was $V = 54V$. Davisson and Germer then calculated the wavelength that the electrons would have from the de Broglie hypothesis. The momentum p can be obtained from the kinetic energy E_k. By definition

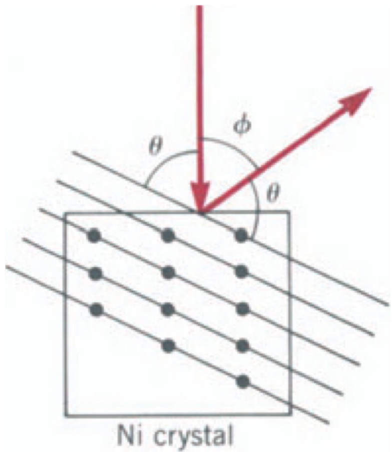

Figure 2: Electrons being scattered by the atomic planes of the nickel crystal.

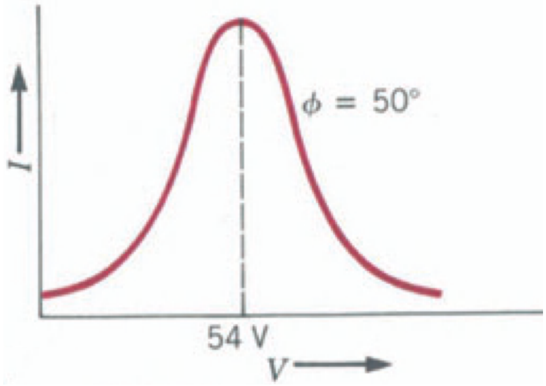

Figure 3: Intensity of the scattered beam, that is, number of scattered electrons, as a function of the accelerating voltage V for a fixed value of the angle $\phi = 50^\circ$.

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}\frac{m^2v^2}{m} \\ &= \frac{p^2}{2m} \\ \Rightarrow p &= \sqrt{2mE_k} \end{aligned}$$

Since $E_k = eV$,

$$p = \sqrt{2meV}$$

Substituting this relation for the momentum into the de Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

they obtained a wavelength for the electrons of

$$\lambda = \frac{6.63 \times 10^{-34} J - sec}{(2 \times 9.1 \times 10^{-31} kg \times 1.6 \times 10^{-19} C \times 54V)^{1/2}} \quad = 1.67 \times 10^{-10} m \quad = 1.67\text{Å}$$

The agreement between the two wavelengths - that is, the predicted wavelength for electrons of this energy and the wavelength of 1.65Å for X rays to be diffracted from nickel - is excellent. Thus the experiment confirms de Broglie's hypothesis that particles have wave properties and that the wavelength of the wave is given by equation (1).