Point Groups and Space Groups

Rotation, refrelction, and inversion are the point operations and their combination give certain symmetry elements which collectively determine the symmetry of space around a point. The group of such symmetry operation at a point is called a **point group**.

In two-dimension space, rotation and reflection are the only point operations. Their combinations yield 10 different point groups designated as 1, 1*m*, 2, 2*mm*, 3, 3*m*, 4, 4*mm*, 6, and 6*mm*. In three-dimensional space, however, the situation is complicated due to the presence of additional point operations such as inversion. There are total of 32 point group in a three-dimensional lattice.

The crystals are classified on the basis of their symmetry which is compared with the symmetry of different point grous. Also, the lattices consistent with the point group operations are limited. Such lattices are known as *Bravais lattices*. These lattices may further be grouped into distinct crystal systems.

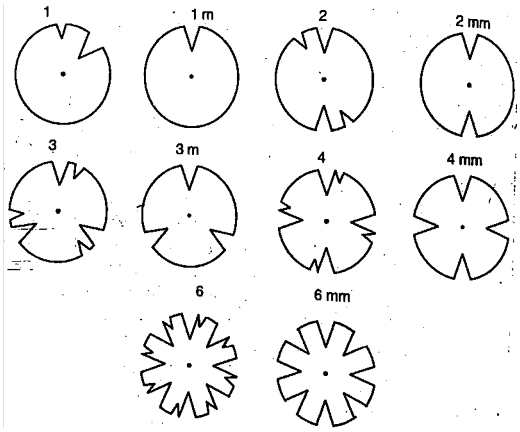

Figure 1: Anisotropy in crystals is due to different arrangement of particles along different directions.

The point symmetry of crystal structures as a whole is determined by the point symmetry of the lattice as well as of the basis. Thus in order to determine the point symmetry of a crystal structure, it should be noted that

1. a unit cell might show point symmetry at more than one locations inside it, and
2. the symmetry element comprising combined point and translation operations might be existing at these locations.

The group of all the symmetry elements of a crystal structure is called *space group*. It determined the symmetry of a crystal structure as a whole. Thre are 17 and 230 distinct space group possible in two and three dimensions respectively.

Types of Lattices

The number of point groups in three dimensions is 32. These point groups from the basis for construction of different types of lattices. Only those lattices are permissible which are consistent wiht the point group operations. Such lattices are called *Bravais lattices*. 32 point groups in three dimensions produce only 14 distinct Bravais lattices. These Bravais lattices further become parts of 7 distinct crystal systems.

All the seven crystal systems of three-dimensional space and corresponding Bravais lattice are listed in Figure.

A lattice point lying at the corner of a cell is shared by eight such cells and the one lying at the face center position is shared by two cells. Therefore, the contribution of the lattice point lying at the corner towards a particular cell is 1/8 and that of a point lying at the face center is 1/2. The effective number of lattice points belonging to a particular cell is given by

$$N = N_i + \frac{N_f}{2} + \frac{N_c}{8}$$

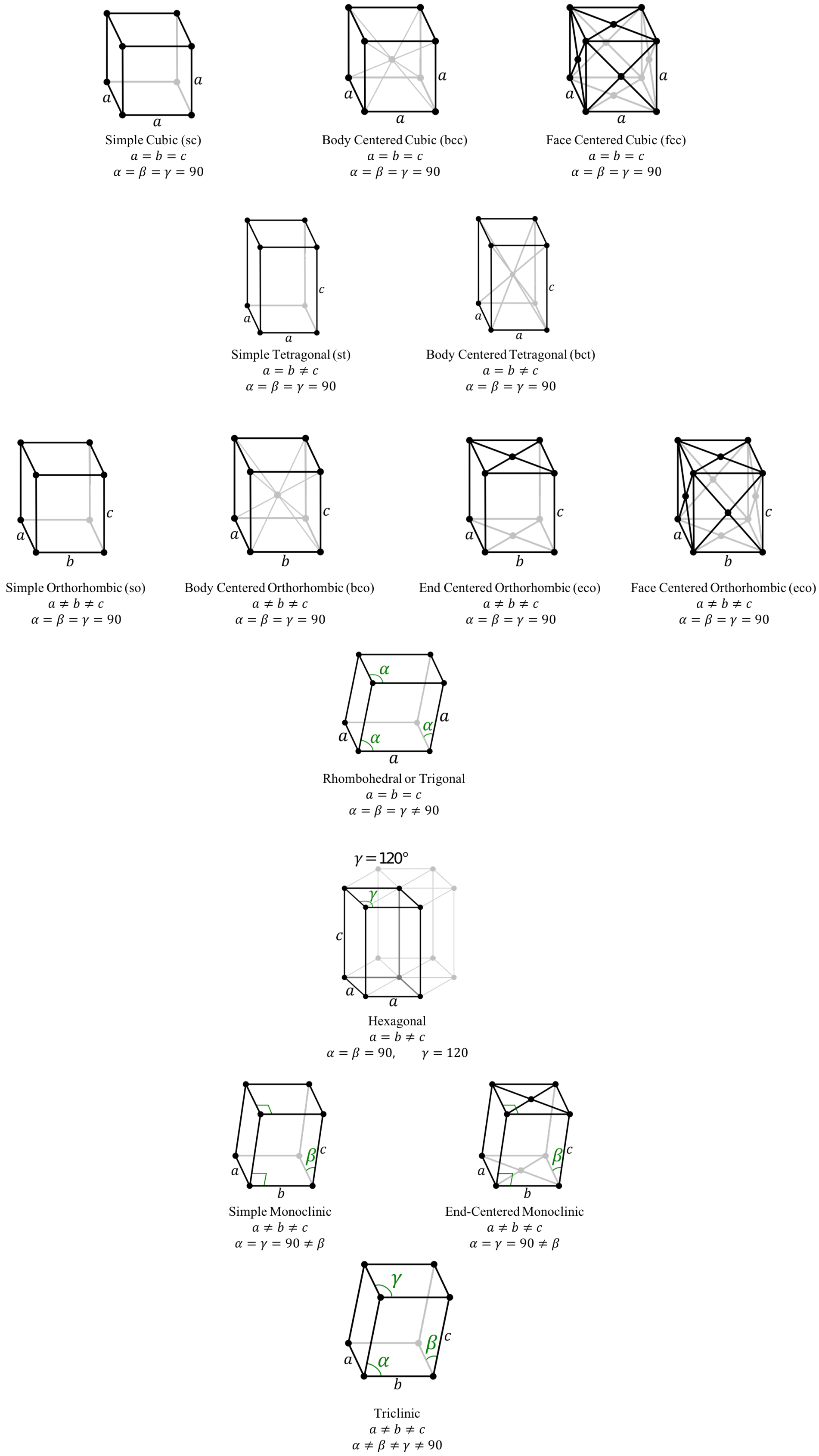

Figure 2: 3d Bravais Lattices