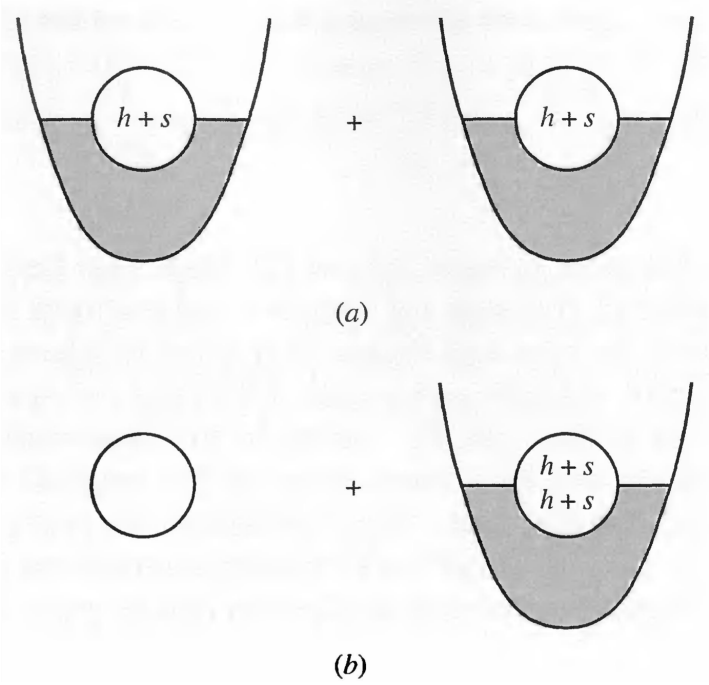

Figure 1: The two electronic systems that are used to compute the pair-binding energy for $n = 1$. Pair binding occurs if the energy of (b) is lower than the energy of (a) . Whether or not this state of affairs obtains for purely electronics reasons depends on the relative magnitudes of the holon-holon, spinon-spinon, and holon-spinon interaction strengths.

1 Pair Binding: Can Electrons do it alone?

It is well known from the early argument of Kohn and Luttinger that any Fermi system has a superconducting instability (albeit at an ultra-low temperature on the order of 10^{-5}K) when electronic vertex corrections are considered. Since their work, there have been numerous arguments presented for purely electronic mechanisms of superconductivity. Rather than present a specific mechanism, we focus instead on a general way of understanding pair binding from purely electronic considerations by appealing to the holon-spinon construct. Let Φ_n be the energy of an electronic system containing n extra electrons. The pair-binding question is as follows: do two isolated systems, each containing $n + 1$ and $n - 1$ extra electrons, have lower total energy than two separate systems with n extra electrons? The energy difference

$$E^(n)_{pair} = 2\Phi_n - \Phi_{n+1} - \Phi_{n-1} \tag{1}$$

is the pair-binding energy. If $E^(n)_{pair} \geq 0$, pairing is expected, as a net attraction favors dispro-

portionation into $n+1$ and $n-1$ states. To illustrate that pair binding can result from repulsive interactions alone, we adopt the spin-charge dichotomy of the Luttinger liquid. For simplicity, we consider the case in which $n = 1$, as illustrated in Fig.(1). To compute the pair-binding energy, we must evaluate $2\Phi_1 - \Phi_2 - \Phi_0$. We set $\Phi_0 = 0$ and let E_h and E_s be the energy of the holon and spinon, respectively. Also, we define V_{hh} and V_{hs} to be the holon-holon and holon-spinon interaction. The relevant physical system is illustrated in Fig.(1). Within an additive constant, the energy of $2\Phi_1$,

$$2\Phi_1 = 2(E_h + E_s + V_{hs}), \tag{2}$$

is simply the sum of the energy of two independent electronic systems with an extra electron added and that of the doubly occupied state is

$$\Phi_2 = 2E_h + V_{hh} + 4V_{hs} + V_{ss} + 2E_s. \tag{3}$$

The factor of 4 arises from the two holon-spinon interactions for each electron and the cross interaction between the two electrons. We have assumed that the spinon energy in the singlet is zero. The energy difference between these states is

$$E^{(1)}_{pair} = -(2V_{hs} + V_{hh} + V_{ss}). \tag{4}$$

Now for an electron to be stable, $V_{hs} \leq 0$. Also, Coulomb's law dictates that $V_{hh} \geq 0$. The spinon-spinon interaction energy then is the crucial quantity that dertermines whether or not $E^{(1)}_{pair} \geq 0$. In some situations, V_{ss} is of the right magnitude, such that $E^{(1)}_{pair} \geq 0$. In this case, pair binding can occur from purely repulsive electron interactions.

2 Excitation Spectrum

If we compare Eq.(**??**) with the standard Hamiltonian for a harmonic oscillator, $H = P^2 + Q^2$, where P and Q are the canonical momentum and position, we find an equivalence between the bosonized charge sector and a collection of harmonic oscillators. As advertised, charge excitations in the Luttinger model are the usual bosonic modes, known more commonly as *plasma excitations.* What is the appropriate density of states for these excitations? It might be suspected that they are governed by the standard Bose-Einstein distribution. However, this is not the case. Charge excitations in a Luttinger liquid have a vanishing density of states at the Fermi level of the form, $\langle k - k_F \rangle^{\angle}$, which is due entirely to the correlations among the electrons. The power law is given by $\angle = (g_c + g_c^{-1})/2 - 1$. Hence, rather than the smooth density of states indicative of Fermi liquids, Luttinger liquids develop a soft gap at the Fermi level for any $U \neq 0$. The source of this effect can be traced to the renormalization of v_F^c by g_c in the bosonized form of the electron gas.

We establish this result by calculating the total charge correlator

$$G(x) = \left\langle \sum_{\sigma} (\Psi_{\sigma^+}(x)\Psi^{\dagger}_{\sigma^+}(0) + \Psi_{\sigma^-}(x)\Psi^{\dagger}_{\sigma^-}(0)) \right\rangle. \tag{5}$$

To compact the notation, we define

$$\Theta_{\sigma}(x) = \int_{-\infty}^{x} \Pi_{\sigma}(x')dx'. \tag{6}$$

For each spin component, we make the substitution $\Phi_{\uparrow(\downarrow)} = (\Phi_c \pm \Phi_s)/\sqrt{2}$ and $\Theta_{\uparrow(\downarrow)} = (\Theta_c \pm \Theta_s)/\sqrt{2}$, where the upper sign applies to up spins and the lower sign to down spins. In the absence of a magnetic field, $n_{\uparrow}(x) = n_{\downarrow}(x)$. As a result, all averages linear in either Φ_s or Π_s vanish identically. We simplify the average in Eq.(5) even further by using the transformed fields $\tilde{\Phi}_{\gamma}(x)$ and $\tilde{\Pi}_{\gamma}(x)$. The average in Eq. (5) will now be over the ground state of the non-interacting bosonized system characterized by the renormalized Fermi velocities, v_F^c and v_F^s. Consequently, we can use the standard rules for simplifying the average of a product of exponentials of boson operators. Using the third form for the Baker-Hausdorff identity (Eq.**??**), we find that the correlator of the total electron density simplifies to

$$G(x) = \frac{1}{\pi a} e^{tphase} e^{\frac{\pi}{2}\sum_{\gamma=c,s}(g_{\gamma}^{-1}a_{\gamma} + g_{\gamma}b_{\gamma})}, \tag{7}$$

where

$$a_{\gamma} = \langle \tilde{\Phi}_{\gamma}(x)\tilde{\Phi}_{\gamma}(0) - \tilde{\Phi^2}_{\gamma}(0) \rangle \tag{8}$$

and

$$b_{\gamma} = \langle \tilde{\Theta}_{\gamma}(x)\tilde{\Theta}_{\gamma}(0) - \tilde{\Theta^2}_{\gamma}(0) \rangle. \tag{9}$$

The phase is determined by the cross terms. For both charge and spin fields, the cross terms are of the form $\langle \tilde{\Phi}_{\gamma}(0)\tilde{\Theta}_{\gamma}(0) + \tilde{Theta}_{\gamma}(0)\tilde{\Phi}_{\gamma}(0) \rangle - \langle \tilde{\Phi}_{\gamma}(x)\tilde{\Theta}_{\gamma}(0) + \tilde{\Theta}_{\gamma}(x)\tilde{\Phi}_{\gamma}(0) \rangle$. This term is identically zero.

The simplest way to evaluate the averages in $G(x)$ is to introduce the Fourier expansion for free boson fields:

$$\begin{aligned}\tilde{\Phi}_\gamma(x) &= \int_{-\infty}^{\infty} \frac{dp}{2\pi\sqrt{2\mid p\mid}} e^{-a|p|/2}[\phi_\gamma(x)e^{\imath px} + \phi_\gamma^\dagger(p)e^{-ipx}] \\ \tilde{\Pi}_\gamma(x) &= \int_{-\infty}^{\infty} \frac{dp\mid p\mid}{2\pi\sqrt{2\mid p\mid}} e^{-a|p|/2}[-i\phi_\gamma(x)e^{ipx} + i\phi_\gamma^\dagger(p)e^{-\imath px}],\end{aligned} \tag{10}$$

where

$$[\phi_\gamma(p), \phi_\gamma^\dagger(q)] = 2\pi\delta(q-p) \tag{11}$$

and $\gamma = c, s$. From these expressions, it follows that

$$\langle\tilde{\Phi}_\gamma(x)\tilde{\Phi}_\gamma(0) - \tilde{\Phi^2}_\gamma(0)\rangle = \frac{1}{4\pi}\ln\left(\frac{a^2}{a^2+x^2}\right). \tag{12}$$

The derivation of the analogous result for the conjugate field is a bit more complicated, and, hence, we perform the calculation explicitly. From the definition of $\tilde{\Theta}(x)$, we reduce the correlator of $\tilde{\Theta}_\gamma(x)$ to

$$\langle\tilde{\Theta}_\gamma(x)\tilde{\Theta}_\gamma(0) - \tilde{\Theta^2}_\gamma(0)\rangle = \int_0^x dx' \int_{-\infty}^0 dx''\langle\tilde{\Pi}_\gamma(x')\tilde{\Pi}_\gamma(x'')\rangle. \tag{13}$$

To simplify the average over the conjugate momentum fields, we note that $\langle\phi_\gamma(p)\phi_\gamma^\dagger(q)\rangle = 2\pi\delta(p-q)$. Consequently,

$$\begin{aligned}\langle\tilde{\Pi}_\gamma(x')\tilde{\Pi}_\gamma(x")\rangle &= \int_{-\infty}^{\infty}\frac{dp\mid p\mid}{4\pi}e^{ip(x'-x")}e^{-a|p|} \\ &= \frac{-1}{2\pi}\left[\frac{(x'-x")^2-a^2}{(a^2+(x'-x")^2)^2}\right].\end{aligned} \tag{14}$$

Substituting this expression into Eq.(13), we find that

$$\begin{aligned}\langle\tilde{\Theta}_\gamma(x)\tilde{\Theta}_\gamma(0) - \tilde{\Theta^2}_\gamma(0)\rangle &= \frac{1}{2\pi}\int_0^x dx' \int_\infty^{x'} dy \frac{y^2-a^2}{(a^2+y^2)^2} \\ &= \frac{-1}{2\pi}\int_0^x dx' \frac{y}{a^2+y^2} = \frac{1}{4\pi}\ln\frac{a^2}{a^2+x^2}\end{aligned} \tag{15}$$

and, hence, the correlator (Eq.7) of the total electron density

$$G(x) \propto \mid x \mid^{-\frac{g_c+g_c^{-1}+g_s+g_s^{-1}}{4}} \tag{16}$$

decays algebraically as a function of distance. Algebraic decay of the electron correlator is the key defining feature of 1d correlated-electron systems. The momentum-distribution function about k_F is defined through the Fourier transform

$$\begin{aligned} n(k) &= \int_{\frac{-L}{2}}^{\frac{L}{2}} dx e^{i(k-k_F)x} G(x) \\ &\propto \int_{\frac{-L}{2}}^{\frac{L}{2}} dx e^{i(k-k_F)x} \mid x \mid^{-\frac{g_c+g_c^{-1}+g_s+g_s^{-1}}{4}} . \end{aligned} \tag{17}$$

Changing variables to $y = (k-k_F)x$, we find that the momentum distribution function for electronic excitations,

$$n(k) \propto \mid k - k_F \mid^{\frac{g_c+g_c^{-1}+g_s+g_s^{-1}}{4}-1}, \tag{18}$$

vanishes algebracially in the vicinity of the Fermi level, as depicted in Fig. (2). For spinless electrons, the factor of $2/\sqrt{2}$ that arises from the transformation to charge and spin fields does not appear. As a result, the factor of $\pi/2$ in the exponent of Eq. (7) is replaced by π. As a consequence, the corresponding exponent for spinless electrons is $(g_c + g_c^{-1})/2 - 1$. In both cases, however, we recover the Fermi-liquid condition that the density of states is a nonzero constant at the Fermi level by setting $g_\gamma = 1$. Algebraic vanishing of the distribution function at the Fermi level signifies that there are no well-defined quasi particles in a Luttinger liquid, unlike the Fermi-liquid case.
The characteristics value of $k-k_F$ at low temperature is given by $\mid k^2/2m - k_F^2/2m \mid \sim \mid k-k_F \mid v_F \sim$ T. Hence, algebraic vanishing of the momentum-distribution function at the Fermi level translates into an algebraic temperature dependence of the form

$$n(T) \propto T^{\frac{g_c+g_c^{-1}}{2}-1} \tag{19}$$

for the excitation spectrum. Such algebraic scaling is expected to have profound experimental consequences.

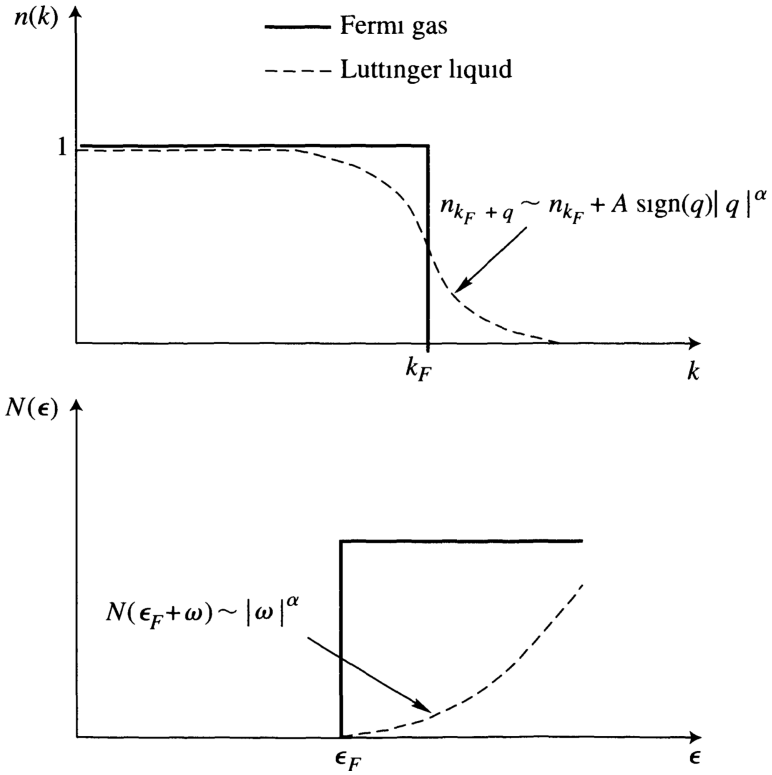

Figure 2: Shown here are the momentum distribution (upper figure) and tunneling density of states (lower figure) for a Luttinger liquid (dashed line) and a Fermi gas (solid line). The characteristic algebraic dependence of the momentum distribution function in the vicinity of the Fermi energy is shown. In the Fermi limit, $\alpha = 0$, and the constant $A = -1/2$.

3 Summary

In 1d, it is possible to develop a bosonized view of the collective excitations of an electron gas in the presence of short-range interactions. This level of description enables a clear demonstration that spin and charge move with fundamentally different velocities in 1d. It is yet unknown how spin and charge separation can be formally established for $d \geq 1$. In addition, Coulomb interactions in 1d give rise to algebraic decay of the density of states at the Fermi level. Such characteristics decay of the density of states is the signature of Luttinger-liquid behavior and is experimentally observable in tunneling experiments.

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