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Vector atom model

The vector atom model basically deals with the total angular momentum of an electron, which is results of the combination of orbital and spin angular momenta. The two fundamental features of the vector atom model are (i) concept of spatial quantization and (ii) spinning electron hypothesis.

1. The concept of spatial quantization
According to Bohr's theory and Sommerfield atomic model the magnitude (shape and size) of orbit is quantized (discrete). But according to quantum theory, the direction or orientation of the orbits in space also should be quantized. To specify the orientation of the electron orbit in space, we need a fixed reference axis. This reference line is chosen as the direction of an external magnetic field that is applied to the atom. The different permitted orientations of an electron orbit are determined by the fact that the projections of the quantized orbits on the field direction must themselves be quantized.

2. Spinning electron hypothesis
According to pinning electron hypothesis, the electron spins about an axis of its own, while it also moved round the nucleus of the atom in its orbit. According to quantum theory, the spin of the electron also should be quantized. Hence a quantum number called *spin quantum number (s)* is introduced.

Since the orbital and spin motions are both quantized in magnitude and direction according to idea of spatial quantization, they are considered as quantized vectors. Hence the atom model based on these quantized vectors is called the "vector atom model".

Quantum number associated with 'Vector atom model'

1. **Principal quantum number** (n)
2. **Orbital quantum number** (l)
3. **Spin quantum number** (s)
4. **Total angular momentum quantum number** (J)
5. **Magnetic orbital quantum number** (m_l)
6. **Magnetic spin quantum number** (m_s)
7. **Magnetic total angular momentum quantum number** (m_j)

1. Principal quantum number
The principal quantum number is used to specify the shape and size of an electron orbit in an atom. Value of principal quantum number is $n = 1, 2, 3, \ldots$.

2. Orbital quantum number (*l*)
Orbital quantum number is used to specify the orbital angular momentum in an atomic model. This can take any integral value $0, 1, 2, 3, \ldots\ldots, (n-1)$. Thus, if $n = 4$, l can take the four values $0, 1, 2, 3$.

By convention, an electron for which $l = 0$ is called s electron, $l = 1$ is called p electron, $l = 2$ is called d electron, $l = 3$ is called f electron.

According to quantum mechanics, the orbital angular momentum of electron is given by

$$p_L = \sqrt{l(l+1)}\,\hbar$$

3. Spin quantum number (*s*)
Spin quantum number $s = 1/2$ is used to indicate the spinning motion of electron in an atom.
According to quantum mechanics the spin angular momentum of electron is given by,

$$p_S = \sqrt{s(s+1)}\,\hbar$$

4. Total angular momentum quantum number (j)
It represents the total angular momentum of the electron which is the sum of the orbital angular momentum and spin angular momentum. It is always positive and is given by

$$j = l \pm s$$

'+' when s is parallel to l and '−' when antiparallel.

The total angular momentum of electron is given by

$$p_J = \sqrt{j(j+1)}\hbar$$

For an electron in $s-orbital$, $l = 0, s = \frac{1}{2}$. Therefore, $j = 0 \pm \frac{1}{2} = \frac{1}{2}$

For an electron in $p-orbital$, $l = 1, s = \frac{1}{2}$. Therefore, $j = 1 \pm \frac{1}{2} = \frac{1}{2}, \ \frac{3}{2}$

5. Magnetic orbital quantum number (m_l)
The projection of orbital quantum number of an electron in an atom along the direction of magnetic is called magnetic orbital quantum number (m_l). The possible values of m_l are

$$m_l = l,\ l-1,\ l-2,\ \dots\dots\ 2,\ 1,\ 0,\ -1,\ -2,\ \dots\dots\ -l$$

Therefore there are $(2l+1)$ possible values of m_l.
When $l = 2$, there are $2l + 1 = 2 \times 2 + 1 = 5$ possible value of m_l. $m_l = -2, -1, 0, 1, 2$
There are 5 orientation of angular momentum vector for $l = 2$. This phenomena of discrete orientation of l vector in space with respect to $\vec{B}$ is known as **space quantization**.

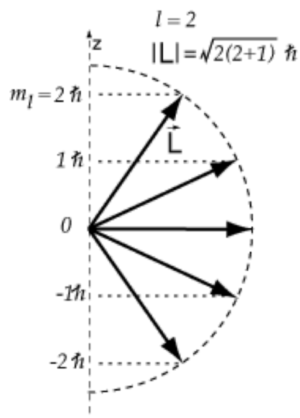

Figure 1: Magnetic orbital quantum numbers for $l = 2$

6. Magnetic spin quantum number (m_s)
This is the projection of spin vector s along the direction of the magnetic field. The spin angular momentum can have only two possible positions with respect to the magnetic field: it may be parallel to it or anti-parallel. m_s can have only two values $+\frac{1}{2}$ or $-\frac{1}{2}$.

7. Magnetic total angular momentum quantum number (m_j)
This is the projection of total angular momentum vector j on the direction of magnetic field. Since we are dealing with one electron, j can have only odd half-integral values ($\because j = l \pm \frac{1}{2}$). Hence m_j must have only odd half-integral values. m_j can have $2j + 1$ values from $+j$ to $-j$.