Absorption of γ rays

Consider a beam of γ-rays is incident on a slab of thickness x. THe beam which emerges from the slab is found to have a smaller intensity of is said to be attenuated. The change in intensity of the beam as it passes through the slab is proportional to the

1. thickness of the slab
2. intensity of the beam

Thus in passing through a small distance dx the change in intensity is

$$dI \propto dx \tag{1}$$

$$dI \propto I \tag{2}$$

combining these two factors,

$$\begin{aligned} dI &\propto Idx \\ &= -\mu Idx \quad \text{where } \mu \text{ is a constant called linear absorption coefficient} \\ \frac{dI}{I} &= -\mu dx \\ \int \frac{dI}{I} &= -\int \mu dx \\ \log_e I &= -\mu x + c \end{aligned}$$

For $x = 0$, $I = I_0$, therefore $c = \log_e I_0$

$$\log_e I = -\mu x + \log_e I_0 \quad \Rightarrow \log_e \left(\frac{I}{I_0}\right) = -\mu x \quad \Rightarrow \frac{I}{I_0} = e^{-\mu x}$$

$$\boxed{I = I_0 e^{-\mu x}} \tag{3}$$

Half value thickness

It is defined as that thickness of the slab for which the intensity of γ- rays reduces to half of its initial intensity.

$$i.e. \quad x_{1/2} = x \quad \text{for} \quad I = \frac{I_0}{2}$$

Therefore,

$$\frac{I_0}{2} = I_0 e^{-\mu\ x_{1/2}} \quad \Rightarrow e^{\mu\ x_{1/2}} = 2 \quad \Rightarrow \mu\ x_{1/2} = \log_e 2$$

$$\boxed{x_{1/2} = \frac{0.693}{\mu}}$$

Compton Scattering

When a beam of γ- rays incident on a material, they suffer a change in wavelength on scattering. When photon of energy $h\nu_i(= h\frac{c}{\lambda_i})$ strikes on electron at rest, the photon with diminished energy $h\nu_f(= h\frac{c}{\lambda_f})$ is scattered at an angle θ with the direction of incident photon and the electron recoils at an angle ϕ. The energy absorbed by these Compton electrons is only a small fraction of the total energy of the incident γ-rays. Using conservation of energy,

$$h\nu_i + m_e c^2 = h\nu_f + \sqrt{p_e^2 c^2 + m_e^2 c^4} \quad \cdots\cdots(1)$$

From conservation of momentum,

$$\vec{p_i} = \vec{p_f} + \vec{p_e} \quad \cdots\cdots(2)$$

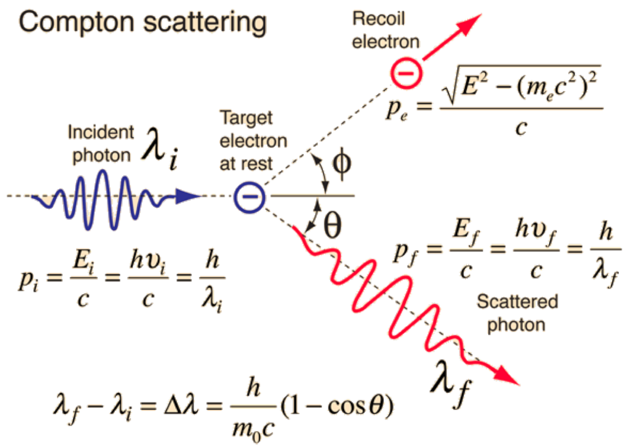


Figure 1: Compton Scattering

Squaring this equation we get,

$$p_e^2 = p_i^2 + p_f^2 - 2p_i p_f \cos\theta \quad \cdots\cdot(3)$$

Multiplying both sides by c^2 and substituting $pc = h\nu$, we get

$$(p_e c)^2 = (h\nu_i)^2 + \left(h\nu_f\right)^2 - 2h^2\nu_i\nu_f \cos\theta \quad \cdots\cdots(4)$$

Squaring equation (1) and rearranging we get,

$$(p_e c)^2 = (h\nu_i)^2 + \left(h\nu_f\right)^2 - 2h^2\nu_i\nu_f + 2m_e c^2\left(h\nu_i - h\nu_f\right) \quad \cdots\cdots(5)$$

Equating equation (4) and (5) we get,

$$-2h^2\nu_i\nu_f\cos\theta = -2h^2\nu_i\nu_f + 2m_e c^2\left(h\nu_i - h\nu_f\right)$$
$$\Rightarrow \frac{1}{h\nu_f} - \frac{1}{h\nu_i} = \frac{1}{m_e c^2}(1-\cos\theta)$$

$$\boxed{\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)} \quad \cdots\cdots(6)$$

Pair Production

When a highly energetic photon incident on a heavy nucleus then the photon can split into a particle and antiparticle pair (electron and positron, for example). This process is called pair production. The pair production happens only when the energy of photon exceeds $2m_0c^2 \approx 1.02MeV$ which is the sum of rest mass energies of electron and positron.

The process of combination of particle and its antiparticle to result a highly energetic photon is called annihilation.

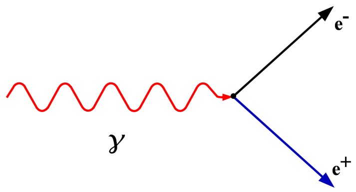


Figure 2: Pair production

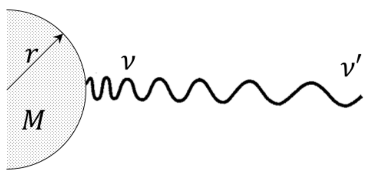
Gravity and the Photon

The relativistic energy expression attributes a mass to any energetic particle, and if applied to the photon with its zero rest mass we can write,

$$m = \frac{h\nu}{c^2}$$

The effective gravitational potential energy of photon emitted from a star of mass M and radius r is then,

$$U = -\frac{GMm}{r} = -\frac{GM\,h\nu}{rc^2}$$



Then the total energy E of the photon is

$$\begin{aligned} E &= h\nu - \frac{GM\,h\nu}{rc^2} \\ &= h\nu\left(1 - \frac{GM}{rc^2}\right) \end{aligned}$$

At a larger distance from the star, the photon is beyond the star's gravitational field but its total energy remains the same. The pnoton's energy is now entirely electromagnetic, and

$$E = h\nu'$$

Hence,

$$h\nu' = h\nu\left(1 - \frac{GM}{rc^2}\right) \quad \Rightarrow \nu' = \nu\left(1 - \frac{GM}{rc^2}\right)$$

Since it is reduced in frequency, this is called the gravitational red shift or the Einstein red shift.

$$\frac{\nu'}{\nu} = 1 - \frac{GM}{rc^2}$$

$$\frac{\Delta\nu}{\nu} = \frac{\nu - \nu'}{\nu} = 1 - \frac{\nu'}{\nu} = \frac{GM}{rc^2}$$