Nuclear Structure

Nuclear composition: Atom consists of a very small nucleus of the size $\approx 10^{-14}m$ surrounded by orbiting electrons. Empirical formula for the nuclear radius is given by

$$R = R_0 A^{\frac{1}{3}} \quad \text{where } R_0 \text{ is a constant } (= 1.3\times 10^{-15}m) \text{ and } A \text{ is the mass number.}$$

A nucleus is made up of elementary particles called protons and neutrons. A proton has positive charge of the same magnitude of that of an electron. A neutron is electrically neutral. The protons and neutrons are collectively called nucleons. The total number of nucleons in a nucleus is called mass number(A). The number of protons is called atomic number. The number of neutron is therefore $A - Z$.

Standard nuclear notation shows the chemical symbol, the mass number and the atomic number of the atom as ${}_ZX^A$. For example ${}_2He^4$ represents nucleus of Helium having 2 protons and 4 nucleons.
Mass of a nucleus is given by sum of mass of all the nucleons.

$$nuclear\ mass = Z\ m_p + (A - Z)\ m_N$$

where m_p is mass a proton and m_N is that of a neutron.

Nuclear density:

$$\begin{aligned}
Nuclear\ density = \frac{Nuclear\ mass}{Nuclear\ volume} &= \frac{Zm_P + (A-Z)m_N}{\frac{4}{3}\pi R^3} \\
&= \frac{Zm_P + Am_P - Zm_P}{\frac{4}{3}\pi R_0^3 A} \quad [\because R = R_0 A^{\frac{1}{3}}\ \&\ m_P \approx m_N] \\
&= \frac{m_P}{\frac{4}{3}\pi\,(1.3\times 10^{-15})^3} = \frac{1.67\times 10^{-27}}{\frac{4}{3}\pi\,(1.3\times 10^{-15})^3} \approx \boxed{2\times 10^{17}kg/m^3}
\end{aligned}$$

Spin angular momentum: Like electrons, protons and neutrons also have intrinsic spin. The spin angular momentum of each nucleon is given as

$$L_s = \sqrt{s(s+1)}\hbar \quad \text{where } s = \frac{1}{2} \qquad \therefore L_s = \frac{\sqrt{3}}{2}\hbar$$

Mass defect and binding energy:
There exists a difference in mass of a nucleus when mass of nucleons are taken separately and the mass of nucleus itself. The diference in mass obtained is called mass defect (Δm).
According to Einstein's mass-energy relationship, the difference in mass changes into energy and this energy is utilized to bind the nucleons in the nucleus to result a stable nucleus.
The energy thus obtained is called binding energy and is given as

$$E = \Delta mc^2$$

The energy required to bind the nucleons together inside the nucleus is called binding energy. In other words, it can also be stated as the minimum energy required to disrupt a stable nucleus into its constituent nucleons is called binding energy.

$$B.E. = \Delta mc^2 \qquad \Rightarrow B.E. = \{(Zm_P + Nm_N) - m\}\,c^2$$

where m is the mass of nucleus.
If $B.E. > 0$, the nucleus is stable and if $B.E. < 0$ the nucleus is unstable.

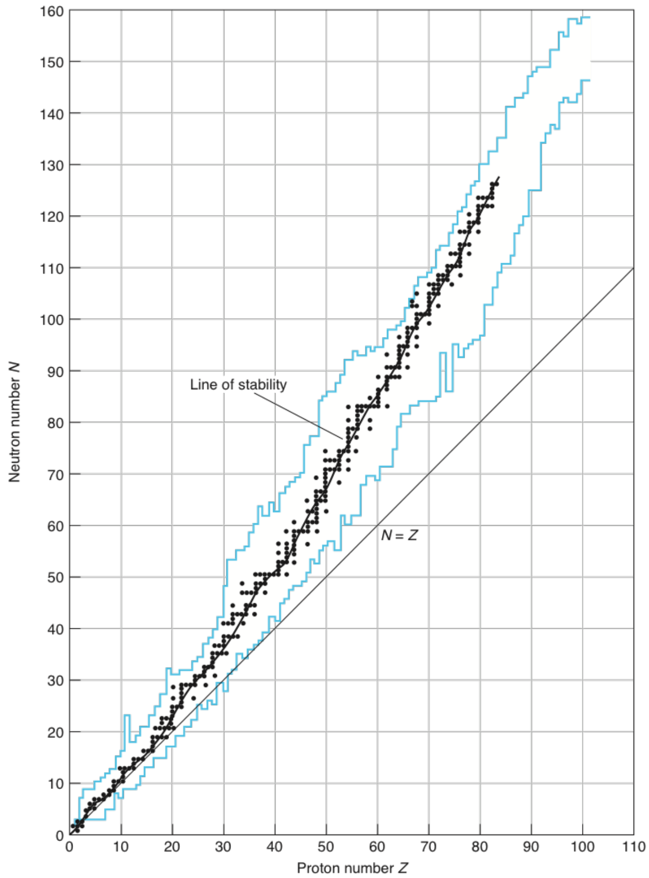

Figure 1: Plot of neutron number N versus proton number Z for the known nuclides. The 257 stable nuclides are indicated by the black dots. The area between the irregular colored lines represents the known unstable, or radioactive, nuclides whose lifetimes are longer than about a millisecond. The curved line through the stable nuclides is called the line of stability.

Stability of a nucleus and binding energy:

Among the more than 3000 known nuclides, there are only about 260 whose ground states are stable. All of the rest have unstable ground states, which eventually undergo radioactive decay, that is, transition to some lower-energy state of a different element. Figure 1 shows a plot of the neutron number N versus the proton number Z for the stable nuclides and the known unstable ones whose lifetimes are longer than about a millisecond. The straight line is $N = Z$.

The binding energy per nucleon is defined as the ratio of the binding energy of nucleus to the number of nucleons present in nucleus.

$$B.E.\ per\ nucleon = \frac{B.E.}{no.\ of\ nucleons}$$

B.E. per nucleon gives the stability of the nucleus.

When the B.E. per nucleon for various element is plotted against the mas number, a curve is obtained as shown in Figure 2. The curve rises very slowly in the begining for lighter nuclei and then rises slowly and reaches to a maximum value ($8.79 MeV$) for an iron nucleus $_{26}Fe^{56}$. The curve then falls down slowly to about $7.6 MeV$ for the heavier nucleus $_{92}U^{238}$. This fact suggests that the nuclei with intermediate mass are more stable. The nuclei with lower mass number and higher mass number are less stable. Hence a

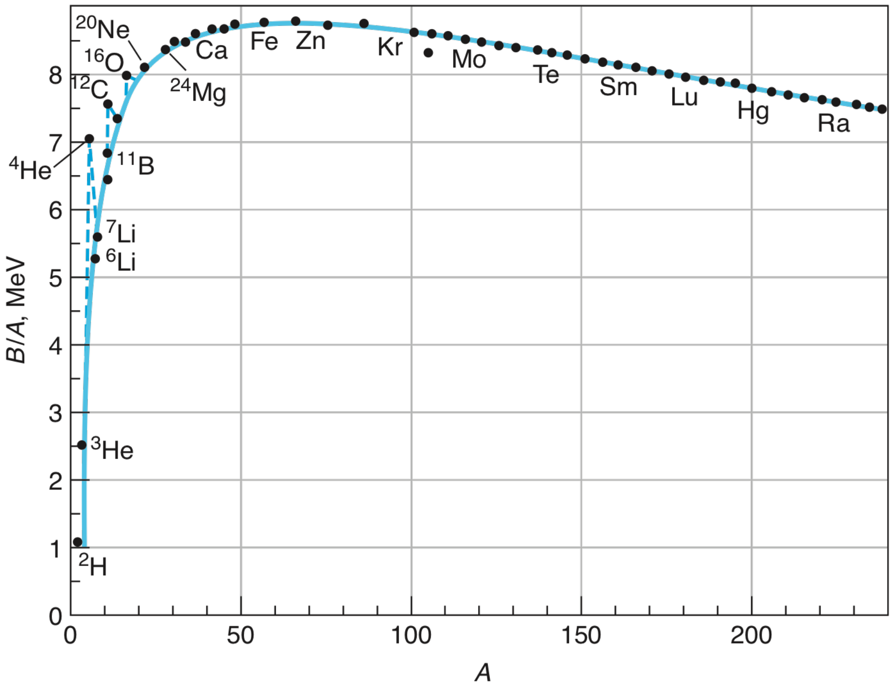

Figure 2: The binding energy per nucleon versus atomic mass number A.

large amount of energy is liberated if heavier nuclei can somehow split into lighter ones. This process is called nuclear fission. Similarly, when two light nuclei unite together to form a nucleus with heavier mass, energy is released, and the process is called nuclear fusion.

Packing fraction

The ratio of mass defect (Δm) to the mass number (A) is called packing fraction. It is denoted by f and is given by

$$f = \frac{\Delta m}{A}$$

Packing fraction means mass defect per nucleon. Packing fraction is used to measure the comparative stability of the atom. Packing fraction can also be expressed as

$$f = \frac{Isotopic\ mass - mass\ no.}{mass\ no.} \times 10^4$$

Packing fraction can be positive or negative. If packing fraction is negative, the isotopic mass is less than the mass number and in that case the mass gets transformed into energy in the formation of that nucleus according to the formula $E = mc^2$. This nuclei is more stable.

A positive packing fraction would mean a tendency towards instability. This is not essentially correct for atoms with lower atomic mass though. It should be noted that the element with mass numbers more than 230 are radioactive and goes through spontaneous disintegration.