Kapil Adhikari

Nuclear Transformation

Radioactivity

The process of spontaneous disintegration of unstable nucleus into stable nucleus with the emission of α-particles, β- particles, and γ- rays is called radioactivity.

The element which exhibit radioactivity are radioactive elements, e.g. Uranium, Thorium, Radium, etc. There are two types of radioactivity:

1. Natural radioactivity

The process of spontaneous emission of radiation from heavy element ($Z > 83$ *or* $A > 206$) occurring in nature is called natural radioactivity. It is not affected by external agents like temperature, pressure, electromagnetic fields, etc.

2. Artificial radioactivity

The process of spontaneous emission of radiation by artificial transmutation of elements is called artificial radioactivity.

Decay scheme for α-particle, β -particles and γ-rays

1) When the nucleus of an atom disintegrates by emitting α particle, its mass number A is reduced by 4 units and atomic number Z by 2 units.

$$_ZX^A \rightarrow \ _{Z-2}Y^{A-4} \ + \ _2He^4 + Q$$

For example,

$$_{92}U^{238} \rightarrow \ _{90}Th^{234} \ + \ _2He^4 + Q$$

2) When an atom disintegrates by emitting β- particle, its mass number A remains same but atomic number Z increases by 1.

$$_ZX^A \rightarrow \ _{Z+1}Y^A \ + \ _{-1}e^0 + Q$$

For example,

$$_6C^{14} \rightarrow \ _7N^{14} \ + \ _{-1}e^0 + Q$$

3) When nucleus emits γ- rays, there is no change in A and Z.

$$\left(_ZX^A\right)^* \rightarrow \ _ZX^A \ + \gamma$$

For example,

$$\left(_{38}Sr^{87}\right)^* \rightarrow \ _{38}Sr^{87} \ + \gamma$$

Laws of radioactive disintegration

1) The radioactivity is a random and spontaneous process and is not affected by external conditions like temperature, pressure, electromagnetic fields, etc.

2) During disintegration each atom emits only one particle (α or β) at a time. This is called displacement law.

3) The rate of disintegration is directly proportional to the no. of atoms present at that instant. This is called decay law.

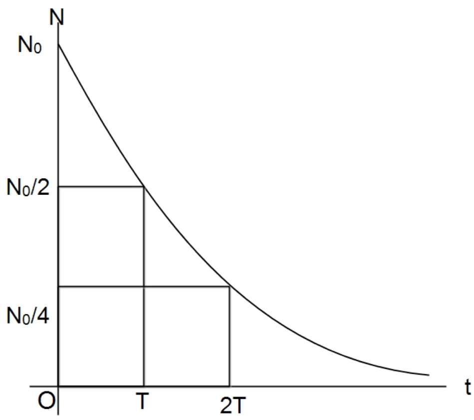

Figure 1: Radioactive decay curve

Decay equation

Let N_0 be the number of atoms present in the radioactive sample at $t = 0$ and N be the number of atoms left after time t. Then, the rate of disintegration, dN/dt is proportional to N.

$$\frac{dN}{dt} \propto N$$
$$\text{or, } \frac{dN}{dt} = -\lambda N$$

Where λ is a constant of proportionality called as disintegration constant or decay constant. The $-$ sign indicates that N decreases as time increases. The number of disintegration per second, dN/dt is called the activity of radioactive sample.

The above equation can be written as

$$\frac{dN}{N} = -\lambda dt$$

integrating on both sides, we get

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$
$$\Rightarrow [\log_e N]_{N_0}^{N} = -\lambda [t]_0^t$$
$$\Rightarrow \log_e N - \log_e N_0 = -\lambda t$$
$$\Rightarrow \log_e \frac{N}{N_0} = -\lambda t$$
$$\Rightarrow N = N_0 e^{-\lambda t}$$

This equation is known as decay equation. It shows that number of active nuclei in a radioactive sample decreases exponentially with time as shown in the figure.

Decay Constant

From the law of radioactive disintegration, we have

$$\frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \lambda = \frac{-\frac{dN}{dt}}{N}$$

Hence the decay constant is defined as the ratio of rate of decay per unit atom present.

If we put t = $\frac{1}{\lambda}$ in decay equation $N = N_0 e^{-\lambda t}$we get,

$$N = N_0 e^{-1} = \frac{N_0}{e}$$

$$= \frac{N_0}{2.718} = 0.37N_0 = 37\% \, of \, N_0$$

Since decay constant may also be defined as the reciprocal of time during which the number of radioactive atoms of a radioactive substance falls to 37% of its original value.

Half Life

The time is taken by a radioactive substance to disintegrate half of its atoms is called the half-life of that substance. It is denoted by $T_{1/2}$ or simply T. Its value is different for different substances.

Relation between half life and decay constant:
Let N_0 be the initial number of atoms in a radioactive substance of decay constant λ . Then after time T, the number of atoms left behind $N_0/2$. So,

$$t = T \text{ and } N = \frac{N_0}{2}$$

Substituting these values in the equation, $N = N_0 e^{-\lambda t}$ we get

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\text{or, } \frac{1}{2} = e^{-\lambda T}$$

$$e^{\lambda T} = 2$$

$$\text{or, } \lambda T = \log_e 2 = 0.693$$

$$T = \frac{0.693}{\lambda}$$

This is the relation between the half-life and decay constant. Thus the half life of the radioactive substance is inversely proportional to its decay constant.

Average Life or Mean Life

The average life or mean life of a radioactive substance is equal to the sum of total life of the atoms divided by the total number of atoms of element.

$$\text{Mean life} = \frac{\text{sum of life of all the atoms}}{\text{total number of atoms}}$$

It can be shown that the mean life of a radioactive substance is equal to the reciprocal of the decay constant.

$$T_{mean} = \frac{1}{\lambda}$$

But $\lambda = \frac{0.693}{T}$ where T is the half life of the substance.

$$\therefore T_{mean} = \frac{T}{0.693} = 1.443T$$

Thus the mean life of a radioactive substance is longer than its half life.

Activity of Radioactive Substance

The rate of decay of a radioactive substance is called the activity (R) of the substance.

$$R = \frac{dN}{dt} = -\lambda N$$
$$\text{or, } |R| = \lambda N = \frac{0.693}{T} N$$

So, $R \propto N$. If R_0 is the activity of a substance at time $t = 0$, then

$$R_0 = \lambda N_0$$
$$\text{or, } \frac{R}{R_0} = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$
$$\text{or, } R = R_0 e^{-\lambda t}$$

Number of atoms left behind after n half lives

Let N_0 be the total number of atoms of a radioactivity substance present at time $t = 0$. Then, the number of atoms present after one half life T is

$$N = \frac{N_0}{2}$$

After two half life, the number present is

$$\frac{1}{2}\frac{N_0}{2} = \left(\frac{1}{2}\right)^2 N_0$$

After n-half life time, the number present is

$$\left(\frac{1}{2}\right)^n N_0$$

Units of Radioactivity

The activity of a radioactive substance is measured in terms of disintegration per second. Following are units of radioactivity.

1) **Curie (Ci)**:
It is defined as the activity of a radioactive substance which gives 3.7×10^{10} integration per second. It is equal to the activity of 1 gm of pure radium.

$$1\text{Ci} = 3.7 \times 10^{10}\text{disintegration/second}$$

2) **Rutherford (rd)**:
It is defined as the activity of radioactive substance which gives rise to 10^6 disintegration per second.

$$1\text{rd} = 10^6\text{disintegration/second}$$

3) **Becquerel (Bq)**:
It is SI-unit of radioactivity.

$$1\text{Bq} = 1\text{disintegration/second}$$

$$1\ \text{Ci} = 3.7 \times 10^{10}\text{disintegration/second} = 3.7 \times 10^{10}\text{Bq} = 3.7 \times 10^4\text{rd}$$